ECL 4340

POWER SYSTEMS

LECTURE 4

POWER TRANSFORMERS

Professor Kwang Y. Lee
Department of Electrical and Computer Engineering
Baylor University

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ANNOUNCEMENT

- Read Chapter 3
- HW 2 is due on Friday, September 9 in Canvas.

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TRANSFORMERS OVERVIEW

- Power systems are characterized by many different voltage levels, ranging from 765 kV down to 240/120 volts.
- Transformers are used to transfer power between different voltage levels.
- The ability to inexpensively change voltage levels is a key advantage of ac systems over dc systems.
- In this section we'll develop models for the transformer and discuss various ways of connecting three phase transformers.

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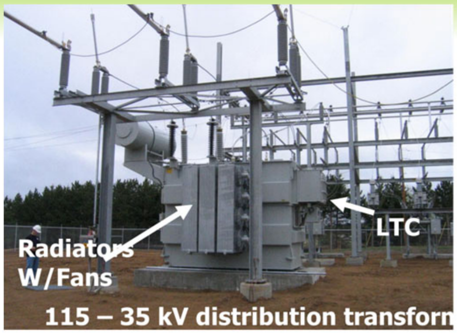
Transmission to Distribution Transformer



LTC is "load tap changer," used for voltage control

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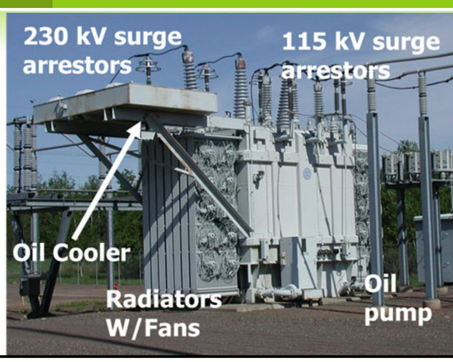
Transmission Level Transformer



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Ideal Transformer

- First, we review the voltage/current relationships for an ideal transformer
  - no real power losses
  - magnetic core has infinite permeability
  - no leakage flux
- We'll define the "primary" side of the transformer as the side that usually takes power, and the "secondary" as the side that usually delivers power.
  - primary is usually the side with the higher voltage but may be the low voltage side on a generator step-up transformer.

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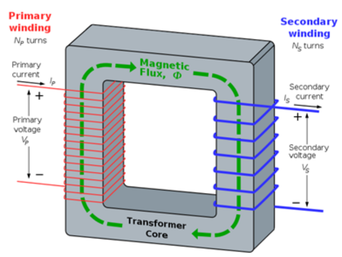
Power Transformers



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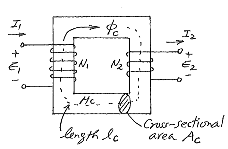
Ideal Transformers

Ideal Transformers:



Assume:

1. zero winding resistance (i^2R loss is zero)
2. infinite permeability ($\mu_c = \infty$, zero reluctance, $\mathcal{R}_c = 0$)
3. no leakage flux
4. no core loss

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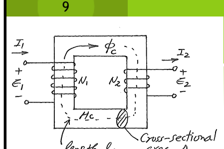
Power Transformers



From Ampere's law,

$$\oint H_{tan}\, dl = I_{enclosed}$$

Tangential component of the magnetic field intensity vector integrated along a closed path equals the net current enclosed by that path.

Assuming the magnetic field intensity H_c is uniform along the closed path of length l_c with cross-sectional area A_c,

$$H_c l_c = N_1 I_1 - N_2 I_2$$

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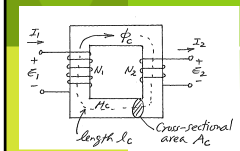
IDEAL TRANSFORMERS



$$H_c l_c = N_1 I_1 - N_2 I_2$$

Note current I_1 is enclosed N_1 times and I_2 is N_2 times. Also, using the right-hand rule, current I_1 produces flux in clockwise direction while current I_2 produces flux in counter-clockwise direction. Thus, the net current enclosed is $(N_1 I_1 - N_2 I_2)$.

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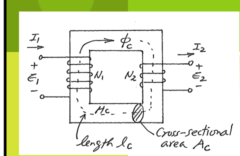
IDEAL TRANSFORMERS



$$H_c l_c = N_1 I_1 - N_2 I_2$$

Flux density B_c and the core flux Φ_c are

$$B_c = \mu_c H_c \quad Wb/m^2$$

$$\Phi_c = B_c A_c \quad Wb$$

where μ_c: permeability of core
A_c: cross-sectional area of core

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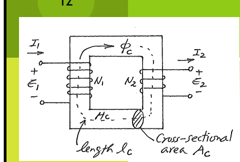
IDEAL TRANSFORMERS



$$H_c l_c = N_1 I_1 - N_2 I_2$$

$$B_c = \mu_c H_c$$

$$\Phi_c = B_c A_c$$

Thus,

$$\underbrace{N_1 I_1 - N_2 I_2}_{\text{mmf}} = \underbrace{\left(\frac{l_c}{\mu_c A_c}\right)}_{\mathcal{R}_c} \Phi_c$$

mmf: magneto motive force; $\mathcal{R}_c$: reluctance

$$\Rightarrow \quad \mathcal{F}_c = \mathcal{R}_c \, \Phi_c$$

: Ohm's law in magnetic circuit

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Ideal Transformers

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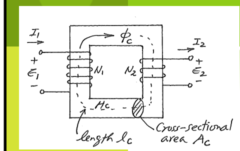

$$\underbrace{N_1 I_1 - N_2 I_2}_{mmf} = \underbrace{\left(\frac{l_c}{\mu_c A_c}\right)}_{\mathcal{R}_c} \Phi_c$$

For ideal transformer, $\mu_c = \infty$, $\Rightarrow$

$$\mathcal{R}_c = \frac{l_c}{\mu_c A_c} = 0$$

Therefore,

$$N_1 I_1 = N_2 I_2$$

or

$$\boxed{\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{a_t}}, \quad a_t = \frac{N_1}{N_2} : \text{turns ratio}$$

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Ideal Transformers

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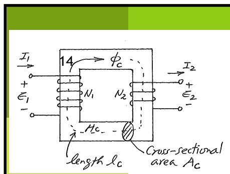

From Faraday's law, voltage (emf, electro motive force) induced across an N-turn winding by a time-varying flux $\phi(t)$ linking the winding is

$$e(t) = N\frac{d\phi}{dt}, \qquad \phi(t) = \Phi_m \cos(\omega t)$$

$$= -\omega N \Phi_m \sin(\omega t)$$

$$= \underbrace{\omega N \Phi_m}_{E_m} \cos(\omega t + 90^\circ)$$

In phasor, $E = \omega N \Phi \angle 90^\circ$

$$= j\omega N \Phi$$

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Ideal Transformers

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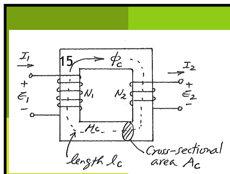

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In phasor, $E = \omega N \Phi \angle 90^\circ$

$$= j\omega N \Phi$$

where Φ and E are phasors for $\phi(t)$ and $e(t)$, respectively.

Note the magnitude of E (in rms) is

$$|E| = \frac{E_m}{\sqrt{2}} = \frac{\omega N \Phi_m}{\sqrt{2}} = \underline{4.44 f N \Phi_m}$$

Thus, the voltage induced is proportional to frequency, number of turns, and the flux in the core.

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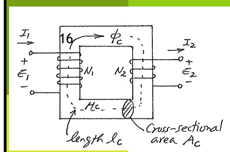


Ideal Transformers

Since the flux Φ_c links both N_1 and N_2 (no leakage flux), the induced voltages in both windings are

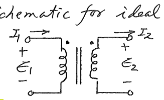
$$E_1 = j\omega N_1 \Phi_c$$

$$E_2 = j\omega N_2 \Phi_c$$

Therefore,

$$\boxed{\frac{E_1}{E_2} = \frac{N_1}{N_2} = a_t}$$

Schematic for ideal transformer:



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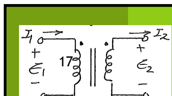

Ideal Transformers

For ideal transformer, power is not lost:

$$S_1 = E_1 I_1^* = (a_t E_2)\left(\frac{I_2}{a_t}\right)^* = E_2 I_2^* = S_2$$

When an impedance Z_2 is connected across winding 2,

$$Z_2 = \frac{E_2}{I_2}$$

The impedance when measured from winding 1, is

$$Z_2' = \frac{E_1}{I_1} = \frac{a_t E_2}{I_2/a_t} = a_t^2 \frac{E_2}{I_2} = \underline{a_t^2 Z_2}$$

Thus, the impedance Z_2 connected to winding 2 (secondary) is referred to winding 1 (primary) by multiplying Z_2 by a_t^2, the square of the turns ratio.

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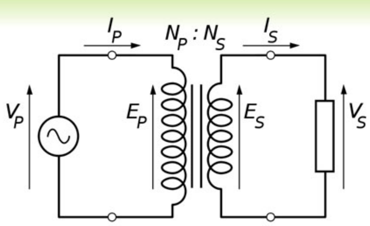
Ideal Transformers



$$\frac{V_P}{V_S} = \frac{I_S}{I_P} = \frac{N_P}{N_S}$$

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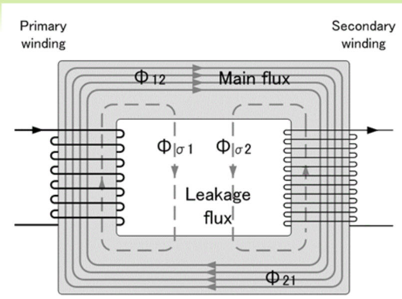
Practical Transformers



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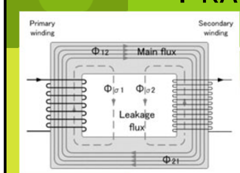
Practical Transformers



Practical Transformers: Now, we remove assumptions made for ideal transformers.

1. Include winding resistances R_1 and R_2 in windings N_1 & N_2.
2. Flux leakages in windings N_1 & N_2. When flux leaks it does not couple to other windings and thus, windings corresponding to the leakage is acting as the usual inductances. We call their reactances leakage reactances X_1 and X_2 for windings N_1 & N_2

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Practical Transformers

3. Permeability is finite. Now $\mathcal{R}_c \neq 0$ (since $\mu_c \neq \infty$).
Recall

$$N_1 I_1 - N_2 I_2 = \mathcal{R}_c \Phi_c \qquad \mathcal{R}_c = \frac{l_c}{\mu_c A_c}$$

or

$$I_1 - \left(\frac{N_2}{N_1}\right) I_2 = \frac{\mathcal{R}_c}{N_1} \Phi_c \quad , \quad E_1 = j\omega N_1 \Phi_c$$

$$= \frac{\mathcal{R}_c}{j\omega N_1^2} E_1$$

$$= -j \underbrace{\left(\frac{\mathcal{R}_c}{\omega N_1^2}\right)}_{B_m,\ \text{susceptance [mhos]}} E_1 \triangleq I_m, \text{ magnetizing current}$$

Thus, the core draws in magnetizing current I_m, which is lagging by 90° to the applied voltage.

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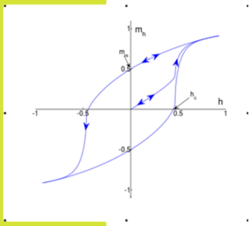
Practical Transformers

4. Core loss is present: There are two types of core loss:
   hysteresis loss and eddy current loss
   - hysteresis loss can be reduced by using high grade alloy steel.



When an external magnetic field is applied to a ferromagnet such as iron, the atomic dipoles align themselves with it. Even when the field is removed, part of the alignment will be retained: the material has become *magnetized*. Once magnetized, the magnet will stay magnetized indefinitely. To demagnetize it requires heat or a magnetic field in the opposite direction. This is the effect that provides the element of memory in a hard disk drive.

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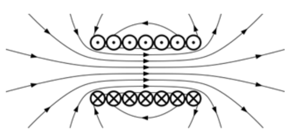
Practical Transformers

4. Core loss is present: There are two types of core loss:
   hysteresis loss and eddy current loss
   - hysteresis loss can be reduced by using high grade alloy steel.
   - eddy current loss can be reduced by using laminated sheets of alloy steel.



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Practical Transformers

Eddy currents

Ferromagnetic materials are also good conductors and a core made from such a material also constitutes a single short-circuited turn throughout its entire length. Eddy currents therefore circulate within the core in a plane normal to the flux and are responsible for resistive heating of the core material. The eddy current loss is a complex function of the square of supply frequency and inverse square of the material thickness. Eddy current losses can be reduced by making the core of a stack of plates electrically insulated from each other, rather than a solid block; all transformers operating at low frequencies use laminated or similar cores.

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Practical Transformers

$$I_1 - \left(\frac{N_2}{N_1}\right) I_2 = \frac{\mathcal{R}_c}{N_1}\Phi_c \quad , \quad E_1 = j\omega N_1 \Phi_c$$

$$= \frac{\mathcal{R}_c}{j\omega N_1^2} E_1$$

$$= \underbrace{-j\left(\frac{\mathcal{R}_c}{\omega N_1^2}\right)}_{B_m,\ \text{susceptance [mhos]}} E_1 \triangleq I_m \text{, magnetizing current}$$

Core loss is present even when the secondary is open. Thus, the core loss current I_c is a shunt current and the core loss is $I_c^2 R_c = I_c^2/G_c = E_1^2 G_c$.

Thus, from (3),

$$I_1 - \left(\frac{N_2}{N_1}\right) I_2 = \underbrace{I_c + I_m}_{I_e\text{ : exciting current}} = (G_c - jB_m) E_1$$

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Practical Transformers

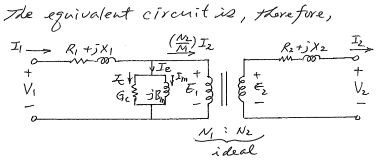
$$I_1 - \left(\frac{N_2}{N_1}\right) I_2 = \underbrace{I_c + I_m}_{I_e\text{ : exciting current}} = (G_c - jB_m) E_1$$

Note: In open circuit, $I_2 = 0$,

$$I_1 = I_c + I_m = I_e$$

This implies that the exciting current is a shunt current

The equivalent circuit is, therefore,



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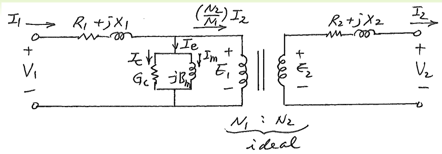
Practical Transformers



Impedances can be moved to either side of the ideal transformer by multiplying the square of the turns ratio. Thus, an equivalent circuit with impedances referred to primary side is

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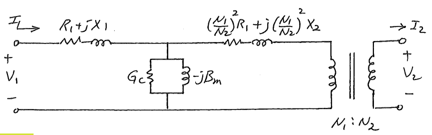
Practical Transformers

Impedances can be moved to either side of the ideal transformer, by multiplying the square of the turns ratio. Thus, an equivalent circuit with impedances referred to primary side is



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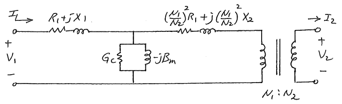
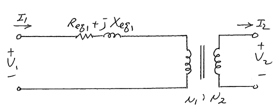
Practical Transformers



By neglecting the exciting current, the equivalent circuit can be further simplified as



where $R_{eq_1} = R_1 + (\frac{N_1}{N_2})^2 R_2$

$X_{eq_1} = X_1 + (\frac{N_1}{N_2})^2 X_2$

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Calculation of Model Parameters

- The parameters of the model are determined based upon
  - nameplate data: gives the rated voltages and power
  - **open circuit test:** rated voltage is applied to primary with secondary open; measure the primary current and losses (the test may also be done applying the voltage to the secondary, calculating the values, then referring the values back to the primary side).
  - **short circuit test:** with secondary shorted, apply voltage to primary to get rated current to flow; measure voltage and losses.

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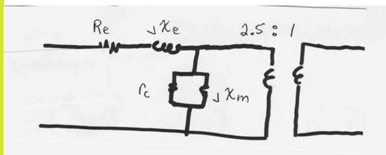
31 Transformer Example

Example: A single phase, 100 MVA, 200/80 kV transformer has the following test data:

open circuit: 20 amps, with 10 kW losses

short circuit: 30 kV, with 500 kW losses

Determine the model parameters.



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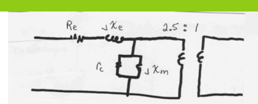
32 Transformer Example, cont'd

A single phase, 100 MVA, 200/80 kV

open circuit: 20 amps, with 10 kW losses

short circuit: 30 kV, with 500 kW losses



From the short circuit test

Rated current in primary winding

$$I_{sc} = \frac{100MVA}{200kV} = 500\ A,\ \left|R_e + jX_e\right| = \frac{30\ kV}{500\ A} = 60\ \Omega$$

$$P_{sc} = R_e I_{sc}^2 = 500\ \text{kW} \rightarrow R_e = 2\ \Omega,$$

$$\text{Hence } X_e = \sqrt{60^2 - 2^2} = 60\ \Omega$$

From the open circuit test

$$R_c = \frac{(200\ kV)^2}{10\ kW} = 4M\Omega$$

$$\left|R_e + jX_e + jX_m\right| = \frac{200\ kV}{20\ A} = 10{,}000\ \Omega \qquad X_m = 10{,}000\ \Omega$$

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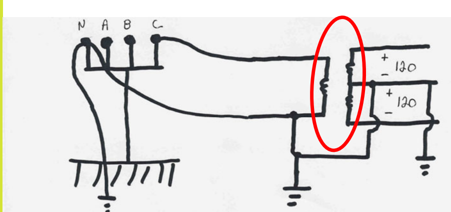
33 Residential Distribution Transformers

Single phase transformers are commonly used in residential distribution systems. Most distribution systems are 4 wire, with a multi-grounded, common neutral.



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Voltage Regulation

Voltage Regulation:

When the load at the secondary is disconnected suddenly, the voltage at the secondary terminal changes abruptly. The amount of such change (in %) is defined as voltage regulation. Formally, it can be written as

$$\% \text{ regulation} = \frac{|V_{2,NL}| - |V_{2,FL}|}{|V_{2,FL}|} \times 100 \quad (\%)$$

where
$V_{2,FL}$: full load (rated) voltage at the secondary
$V_{2,NL}$: no load (open-circuit) voltage at the secondary.

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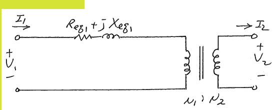

When the secondary is open, $I_2 = 0$. Therefore, $I_1 = 0$. Then, V_1 is applied directly to the N_1 winding of the ideal transformer and V_2 is the voltage induced on the N_2 winding:

$$V_{2,NL} = \left(\frac{N_2}{N_1}\right) V_1$$

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